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A Novel Multi-Criteria Decision Making Method for Evaluating Water Reuse Applications under Uncertainty

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Abstract

There are currently many places in the world where water is scarce. Therefore, water reuse has been mentioned by many researchers. Evaluation of water reuse applications is the selection of the best water reuse application of the existing options; it is also one of the applications of multi-criteria decision making (MCDM). In this paper, we introduce a new dissimilarity measure of picture fuzzy sets. This new measure overcomes the restriction of other existing dissimilarity measures of picture fuzzy sets. Then, we apply it to the multi-criteria decision making. Finally, we refer to a new method for selecting the best water reuse application of the available options by using the picture fuzzy MCDM.

Keywords

Multi-criteria decision making, picture fuzzy, water reuse

Introduction

Reuse of water refers to the treatment and rehabilitation of non-traditional or deteriorated water for beneficial purposes (Miller, 2006). Water reuse is synonymous with using reclaimed water, which can provide an option to reduce water scarcity, especially under the new reality of climate change and the increase in human activities. Water reuse has become widespread all over the world to solve the depletion of water resources, leading to reduced water supplies. Evaluation of water reuse applications is a weight replacement process and the most appropriate selection of water reuse applications. From this, the assessment involves analyzing many criteria with social, technical, economic, political, environmental, and technical aspects to ensure sustainable decision making (Zarghami and Szidarovszky, 2009). The challenge with water reuse application evaluation (WRAE) is that alternatives are diverse in nature, and often have conflicting criteria. The fuzzy set theory (Zadeh, 1965) is a very effective method for solving such contradictory and uncertain problems.

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Fuzzy set theory was introduced by Zadeh in 1965. Immediately, it became a useful method to study the problems of imprecision and uncertainty. Since then, many new theories treating imprecision and uncertainty have been introduced. For instance, an intuitionistic fuzzy set was introduced in 1986 (Atanassov, 1986), which is a generalization of the notion of a fuzzy set. While fuzzy set gives the degree of membership of an element in a given set, the intuitionistic fuzzy set gives a degree of membership and a degree of non-membership. Picture fuzzy set (Cuong and Kreinovich, 2013) is an extension of the crisp set, fuzzy set, and intuitionistic set. A picture fuzzy set has three memberships: a degree of positive membership, a degree of negative membership, and a degree of neutral membership of an element in this set. This approach is widely used by researchers in both theory and application. Hoa and Thong (2017) improved fuzzy clustering algorithms using picture fuzzy sets and applications for geographic data clustering. Son (2015) and Son (2017) presented an application of picture fuzzy set in the problem of clustering. Dinh *et al.* (2015) introduced the picture fuzzy database and examples of using the picture fuzzy database. Dinh *et al.* (2017) investigated distance measures and dissimilarity measures on picture fuzzy sets and applied them in pattern recognition. But these dissimilarity measures of Dinh *et al.* (2017) have a restriction that is further explored in the next section.

We often use decision making methods because of the uncertainty and complexity of the nature of decision making. By the multi-criteria decision making (MCDM) methods, we can determine the best alternative from multiple alternatives for a set of criteria. In recent times, the choice of suppliers has increasingly played an important role in both academia and industry. Therefore, there are many MCDM techniques developed for the supplier selection (Bhutia and Phipon, 2012; Jadidi *et al.*, 2010; Yildiz and Yayla, 2015). However, the above methods have limited use in set theory. Therefore, it is difficult to encounter problems of uncertain or incomplete data. There are several authors who have proposed MCDM methods using fuzzy set theory or intuitionistic fuzzy set for the supplier selection (Boran *et al.*, 2009; Kavita *et al.*, 2009; Yayla, 2012; Maldonado-Macías *et al.*, 2014; Pérez *et al.*, 2015; Omorogbe, 2016; Solanki *et al.*, 2016; Zeng and Xiao, 2016).

With the considered criteria for water reuse applications (Pan *et al.*, 2018), there are usually three levels. For example, the public acceptability attribute has three levels: agreement, disagreement, and neutrality; here we consider the level of agreement as the degree of positive membership, level disagreement as the degree of negative membership, and level neutrality as the degree of neutral membership of the criteria of public acceptability in each alternative. Therefore, we use the multi-criteria decision making method based on picture fuzzy set to select the best alternative in evaluating water reuse applications.

In this paper, we propose a new dissimilarity measure of picture fuzzy sets. This measure overcomes the restriction of the four dissimilarity measures of picture fuzzy sets introduced by Dinh *et al.* (2017). We then propose a MCDM based on the new dissimilarity measure and apply it for evaluating the water reuse applications under uncertainty.

The rest of the paper is organized as follows: In the next section, we recall the concept of picture fuzzy set and several operators of two picture fuzzy sets. We then propose a new MCDM method using the dissimilarity measure of picture fuzzy sets. Finally, we apply the proposed method for evaluating water reuse applications.

Preliminaries

Picture fuzzy sets

Definition 1 (Cuong and Kreinovich, 2013). Let U be a universal set. A picture fuzzy set (PFS) A on the U is $A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u)) | u \in U\}$ where $\mu_A(u)$ is called the "degree of positive membership of u in A", *ηAx(∈ 0,1)* is called the "degree of neutral membership of u in A", and $\gamma_A(u)$γAx(∈ 0,1) is called the "degree of negative membership of u in A" where $\mu_A(u)$, $\eta_A(u)$, and μA,γA$\gamma_A(u) \in [0,1]$ ηAsatisfy the following condition:

$0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1, \forall u \in U$.

The family of all picture fuzzy sets in U is denoted by PFS(U).

For convenience in this paper, we call P is a picture fuzzy number where $P = (a, b, c)$ in which $a, b, c \geq 0$ and $a + b + c \leq 1$.

Definition 2 (Cuong and Kreinovich, 2013). The picture fuzzy set $B = \{(u, \mu_B(u), \eta_B(u), \gamma_B(u)) | u \in U\}$ is called the subset of the picture fuzzy set $A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u)) | u \in U\}$ iff $\mu_B(u) \leq \mu_A(u), \eta_B(u) \leq \eta_A(u)$ and $\gamma_B(u) \geq \gamma_A(u)$ for all $u \in U$.

Definition 3 (Cuong and Kreinovich, 2013). The complement of picture fuzzy set $A = \{(u, \mu_A(u), \eta_A(u), \gamma_A(u)) | u \in U\}$ is

$A^C = \{(u, \gamma_A(u), \eta_A(u), \mu_A(u)) | u \in U\}$.

Definition 4 (Cuong and Kreinovich, 2013). Let A, B be two picture fuzzy sets on U. Then

$A \cup B = \{(u, \max\{\mu_A(u), \mu_B(u)\}, \min\{\eta_A(u), \eta_B(u)\}, \min\{\gamma_A(u), \gamma_B(u)\}) | u \in U\}$ and

$A \cap B = \{(u, \min\{\mu_A(u), \eta_B(u)\}, \min\{\eta_A(u), \eta_B(u)\}, \max\{\gamma_A(u), \gamma_B(u)\}) | u \in U\}$.

New dissimilarity measure of picture fuzzy sets

Firstly, we recall the concept of dissimilarity measure for picture fuzzy sets:

Definition 5 (Dinh *et al.*, 2017). A function $DIS: PFS(U) \times PFS(U) \rightarrow [0,1]$ is a dissimilarity measure between PFS-sets if it satisfies the following properties:

PF-Diss 1: $DIS(A, B) = DIS(B, A)$;

PF-Diss 2: $DIS(A, A) = 0$;

PF-Diss 3: If $A \subset B \subset C$ then $DIS(A, C) \geq \max\{DIS(A, B), DIS(B, C)\}$.

Now, we propose the new dissimilarity measure for picture fuzzy sets:

Definition 6: Let $U = \{u_1, u_2, \ldots, u_N\}$ be the universe set. Let w_i be the weight of element u_i of U in which $0 \leq w_i \leq 1$ and $\sum_{i=1}^{N} w_i = 1$. Given two picture fuzzy sets $A = \{(u_i, \mu_A(u_i), \eta_A(u_i), \gamma_A(u_i)) | u_i \in U\}$ and $B = \{(u_i, \mu_B(u_i), \eta_B(u_i), \gamma_B(u_i)) | u_i \in U\}$, we denote

$DIS_E(A, B) = \sum_{i=1}^{N} w_i DIS_E^i(A, B)$ (1)

where

$DIS_E^i(A, B) = \frac{1 - e^{-|\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|}}{3}$ $(i = 1, 2, \ldots, N)$.

Theorem 1: The formula $DIS_E(A, B)$ determined in Eq.(1) is a dissimilarity measure of two picture fuzzy sets A and B.

Proof.

We have $0 \leq \mu_A(u_i), \mu_B(u_i), \eta_A(u_i), \eta_B(u_i), \gamma_A(u_i), \gamma_B(u_i) \leq 1$ for all $i = 1, 2, \ldots, N$. Hence, $0 \leq DIS_E^i(A, B) \leq 1$ for all $i = 1, 2, \ldots, N$. This implies that $0 \leq DIS_E(A, B) \leq 1$.

It is easily verified that:

+ PF-Diss 1: $DIS(A, B) = DIS(B, A)$;

+ PF-Diss 2: $DIS(A, A) = 0$;

+ With PF-Diss 3, if $A \subset B \subset C$ we have

$$\begin{cases} \mu_A(u_i) \leq \mu_B(u_i) \leq \mu_C(u_i) \\ \eta_A(u_i) \leq \eta_B(u_i) \leq \eta_C(u_i) \\ \gamma_A(u_i) \geq \gamma_B(u_i) \geq \gamma_C(u_i) \end{cases}$$

for all $u_i \in U$.

So that, we have

$\max\{|\mu_B(u_i) - \mu_A(u_i)|, |\mu_C(u_i) - \mu_B(u_i)|\} \le |\mu_A(u_i) - \mu_C(u_i)|$,
$\max\{|\eta_B(u_i) - \eta_A(u_i)|, |\eta_C(u_i) - \eta_B(u_i)|\} \le |\eta_A(u_i) - \eta_C(u_i)|$,
and
$\max\{|\gamma_B(u_i) - \gamma_A(u_i)|, |\gamma_C(u_i) - \gamma_B(u_i)|\} \le |\gamma_A(u_i) - \gamma_C(u_i)|$
for all $u_i \in U$.
It is also implies that
$\max\{1 - e^{-|\mu_B(u_i)-\mu_A(u_i)|}, 1 - e^{-|\mu_C(u_i)-\mu_B(u_i)|}\} \le 1 - e^{-|\mu_A(u_i)-\mu_C(u_i)|}$,
$\max\{|\eta_B(u_i) - \eta_A(u_i)|, |\eta_C(u_i) - \eta_B(u_i)|\} \le |\eta_A(u_i) - \eta_C(u_i)|$,
and
$\max\{|\gamma_B(u_i) - \gamma_A(u_i)|, |\gamma_C(u_i) - \gamma_B(u_i)|\} \le |\gamma_A(u_i) - \gamma_C(u_i)|$
for all $u_i \in U$.
This means that $\max\{DIS_E^i(A,B), DIS_E^i(B,C)\} \le DIS_E^i(A,C)$ for all $u_i \in U$.
This leads to $\max\{DIS_E(A,B), DIS_E(B,C)\} \le DIS_E(A,C)$.

Comparisons to existing dissimilarity measures of picture fuzzy sets

In this section, we compare the new dissimilarity measure with several existing dissimilarity measures of picture fuzzy sets.

Given $U = \{u_1, u_2, \ldots, u_n\}$ is an universe set. Given two picture fuzzy sets $A, B \in PFS(U)$. We have some dissimilarity measures of the picture fuzzy sets (Dinh *et al*., 2017):

$$DM_C(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[|S_A(u_i) - S_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)|] \quad (2)$$

where $S_A(u_i) = |\mu_A(u_i) - \gamma_A(u_i)|$ and $S_B(u_i) = |\mu_B(u_i) - \gamma_B(u_i)|$.

$$DM_H(A,B) = \frac{1}{3n}\sum_{i=1}^{n}[|\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|] \quad (3)$$

$$DM_L(A,B) = \frac{1}{5n}\sum_{i=1}^{n}[|S_A(u_i) - S_B(u_i)| + |\mu_A(u_i) - \mu_B(u_i)| + |\eta_A(u_i) - \eta_B(u_i)| + |\gamma_A(u_i) - \gamma_B(u_i)|] \quad (4)$$

$$DM_O(A,B) = \frac{1}{\sqrt{3n}}\sum_{i=1}^{n}[|\mu_A(u_i) - \mu_B(u_i)|^2 + |\eta_A(u_i) - \eta_B(u_i)|^2 + |\gamma_A(u_i) - \gamma_B(u_i)|^2]^{\frac{1}{2}} \quad (5)$$

These measures have a restriction, which is shown in the following example:

Example 1. Assume that there are two patterns denoted by picture fuzzy sets on $U = \{u_1, u_2\}$ as follows:

$A_1 = \{(u_1, 0,0,0), (u_2, 0.1,0,2,0.1)\}$ and
$A_2 = \{(u_1, 0,0,0.1), (u_2, 0.2,0.2,0.1)\}$.
Now, there is a sample $B = \{(u_1, 0,0.1,0.1), (u_2, 0.1,0.1,0.1)\}$.

Question: Which class of patterns does B belong to?

Using four dissimilarity measures in the Eq.(2), Eq.(3), Eq.(4), and Eq.(5) we have
+ $DM_C(A_1,B) = DM_C(A_2,B) = 0.05$,
+ $DM_L(A_1,B) = DM_L(A_2,B) = 0.04$,
+ $DM_H(A_1,B) = DM_H(A_2,B) = 0.05$, and
+ $DM_O(A_1,B) = DM_O(A_2,B) = 0.0986$.

We can easily see that B does not belong to the class of pattern A_1 or the class of pattern A_2.
Meanwhile, if using the new dissimilarity measure in Eq.(1) then we have
$DM_C(A_1,B) = 0.05, DM_C(A_2,B) = 0.0491$.

We can easily see that sample B belongs to the class of pattern A_2.

This example shows that our proposed dissimilarity measure has overcome the restriction of four dissimilarity measures of picture fuzzy sets which was introduced by Dinh *et al.* (2017).

The proposed MCDM method

In this section, we propose a new method for multi-criteria decision making problems using the new dissimilarity measure of picture fuzzy sets. The multi-criteria decision making problem is determined to be the best alternative from the concepts of the compromise solution. The best compromise solution is the alternative which obtains the smallest dissimilarity measure from each alternative to the perfect choice. The procedures of the proposed method can be expressed as follows.

Input: Let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ be the set of criteria with the weight of each criteria C_j is w_j where $j = 1,2,\ldots,n$ and $\sum_{j=1}^{n} w_j = 1$. For each alternative, A_i $(i = 1,2,..,m)$ is a picture fuzzy set on C, which means that:

$$A_i = \left\{\left(C_j, d_{ij}^1, d_{ij}^2, d_{ij}^3\right) | C_j \in C\right\}.$$

The picture fuzzy decision making matrix $D = (d_{ij})$ in which $d_{ij} = \left(d_{ij}^1, d_{ij}^2, d_{ij}^3\right)$ is a picture fuzzy number for all $j = 1,2,\ldots,n$ and $i = 1,2,\ldots,m$ is as follows:

$$\begin{array}{c} D \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{pmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ d_{m1} & d_{m2} & \ldots & d_{mn} \end{pmatrix} \end{array}$$

Output: Ranking of alternatives

The proposed method is presented with the following steps.

Step 1. Normalizing the decision matrix

In this step, we construct the picture fuzzy decision making matrix. For instance, the j_th column of the decision making matrix is the natural number (but does not form the picture fuzzy number)

$$\begin{array}{c} C \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} C_j \\ \begin{pmatrix} c_{1j}^1 & c_{1j}^2 & c_{1j}^3 \\ c_{2j}^1 & c_{2j}^2 & c_{2j}^2 \\ \vdots & \vdots & \vdots \\ c_{mj}^1 & c_{mj}^2 & c_{mj}^3 \end{pmatrix} \end{array}$$

where $c_{ij}^k > 0$ for all $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n; k = 1,2,3$. We will calculate

$$\begin{array}{c} C \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} C_j \\ \begin{pmatrix} c_{1j}^1 & c_{1j}^2 & c_{1j}^3 \\ c_{2j}^1 & c_{2j}^2 & c_{2j}^2 \\ \vdots & \vdots & \vdots \\ c_{mj}^1 & c_{mj}^2 & c_{mj}^3 \end{pmatrix} \end{array} \xrightarrow[d_{ij}^k = \frac{c_{ij}^k}{\sum_{k=1}^{3} c_{ij}^k}]{} \begin{array}{c} D \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} D_j \\ \begin{pmatrix} d_{1j}^1 & d_{1j}^2 & d_{1j}^3 \\ d_{2j}^1 & d_{2j}^2 & d_{2j}^2 \\ \vdots & \vdots & \vdots \\ d_{mj}^1 & d_{mj}^2 & d_{mj}^3 \end{pmatrix} \end{array}. \quad (6)$$

Then $D = (d_{ij})$ in which $d_{ij} = \left(d_{ij}^1, d_{ij}^2, d_{ij}^3\right)$ is a picture fuzzy decision making matrix.

This step is ignored if matrix D is the given picture fuzzy decision making matrix.

Step 2. Determining the weight of each criteria

We determine the weight w_j $(j = 1,2, \dots, n)$ of the criteria C_j $(j = 1,2, \dots, n)$ such that $\sum_{j=1}^{n} w_j = 1$.

For instance $w_j = \frac{d_j}{\sum_j^n d_j}$ (7)

where $d_j = d_{1j} + d_{2j} + d_{3j}$ and $d_{1j} = \max_{i=1,2,\dots,m} d_{ij}^1$, $d_{2j} = \min_{i=1,2,\dots,m} d_{ij}^2$, $d_{3j} = \min_{i=1,2,\dots,m} d_{ij}^3$ for all $j = 1,2,\dots,n$.

Note that (d_{1j}, d_{2j}, d_{3j}) $(j = 1,2, \dots, n)$ are picture fuzzy numbers.

Step 3. Determining the perfect choice

In this section, we determine the perfect choice. Here, we pay attention to the benefit criteria and cost criteria. Usually, with the perfect choices, we can take the picture fuzzy number (1,0,0) for the benefit criteria and (0,0,1) for the cost criteria. Note that (1,0,0) is the largest value of a picture fuzzy linguistic and (0,0,1) is the smallest value of a picture fuzzy linguistic. Thus, the perfect choice A_b gets the picture fuzzy number $A_b(j)$ at the criteria C_j, in which $A_b(j) = (1,0,0)$ if C_j is the benefit criteria and $A_b(j) = (0,0,1)$ if C_j is the cost criteria, for all $j = 1,2, \dots, n$.

Step 4. Calculating the dissimilarity measure of each alternative to the perfect choice

From Eq.(1) we have the dissimilarity measure of each alternative and the perfect choice which are calculated by

$$DIS_E(A_i, A_b) = \sum_{j=1}^{n} w_j DIS_E^j(\mathrm{A_i}, \mathrm{A_b}), i = 1,2, \dots, m \qquad (8)$$

Step 5. Ranking the alternatives

Now, we can rank the alternatives based on the dissimilarity measure of the each alternative and the perfect choice as follows

$$A_{i_1} \prec A_{i_2} \text{ iff } DIS(A_{i_1}, A_b) > DIS(A_{i_2}, A_b) \qquad (9)$$

$$A_{i_1} \simeq A_{i_2} \text{ iff } DIS(A_{i_1}, A_b) = DIS(A_{i_2}, A_b).$$

The proposed method for evaluating water reuse applications

In this section, we use our proposed method presented in section 3 to evaluate water reuse applications. The data were taken from Pan *et al.* (2018). The problem is as follows. There are seven alternative water reuse systems, namely A_1: toilet flushing (TF); A_2: vegetable watering in gardens (VW); A_3: flower watering in gardens (FW); A_4: agricultural irrigation (AI); A_5: public parks watering (PPW); A_6: golf course watering (GCW); and A_7: drinking water (DW). We need to determine the best option based on five specific criteria, namely C_1: public acceptability (PA); C_2: freshwater saving (FS); C_3: life cycle cost (LCC); C_4: human health risk (HHR); and C_5: the local governments' polices (GP).

The criteria data for public acceptability, freshwater saving, life cycle cost and human health risk were collected as positive real numbers. Data for the governments' policies was given in the form of linguistic variables. All the collected data are shown in Tables 1 and 2. The value picture fuzzy numbers of the linguistic variables are shown in Table 3.

We consider that C_1, C_2, C_5 are the benefit criteria and C_3, C_4 are the cost criteria.

Now, we present the process of our method for evaluating the water reuse applications.

Step 1. Normalizing the decision matrix

From Eq.(6), we obtain the normalization decision matrix (Table 4).

Table 1. Public acceptability and freshwater saving data

Alternatives	C_1: public acceptability			C_2: freshwater saving (ML/year)		
	Agreement	Neutrality	Disagreement	Low	Mid	High
TF (A_1)	80	9	11	428.8	536	643.2
VW (A_2)	63.5	13	23.5	2624.8	3281	3937.2
FW (A_3)	84.5	10	5.5	3192.5	3990.6	4788.8
AI (A_4)	74.5	10	15.5	3192.5	3990.6	4788.8
PPW (A_5)	85.5	8	6.5	886.3	1107.9	1329.5
GCW (A_6)	88.5	7	4.5	361.8	452.3	542.7
DW (A_7)	24	14	62	3192.5	3990.6	4788.8

Table 2. Life cycle cost, human health risk, and government policies data

Alternatives	C_3: life cycle cost (USD/year)			C_4: human health risk (DALY/capita/year)			C_5: governments' policies
	Low	Mid	High	Low	Mid	High	
TF (A_1)	1555358	1944198	2333038	7.10E-12	7.51E-12	8.30E-12	M (Moderate)
VW (A_2)	1637219	2046524	2455829	1.83E-11	1.89E-11	2.03E-11	L (Low)
FW (A_3)	834019	1042524	1251028	1.78E-11	1.84E-11	1.99E-11	H (High)
AI (A_4)	146660	183326	219991	9.07E-12	1.00E-11	1.26E-11	M (Moderate)
PPW (A_5)	635529	794411	953293	9.34E-12	9.77E-12	1.07E-11	H (High)
GCW (A_6)	78219	97774	117328	8.43E-12	8.87E-12	9.83E-12	M (Moderate)
DW (A_7)	1197674	1497092	1796511	2.76E-08	4.01E-08	1.00E-07	VL (Very low)

Table 3. The picture fuzzy number of linguistic variables

Linguistic variables	Picture fuzzy number
M	(0.5,0.4,0.1)
L	(0.2,0.5,0.3)
H	(0.8,0.1,0.05)
M	(0.5,0.4,0.1)
H	(0.8,0.1,0.05)
M	(0.5,0.4,0.1)
VL	(0.1,0,0.9)

Table 4. Decision matrix

	C_1	C_2	C_3
A_1	(0.8,0.09, 0.11)	(0.266667,0.333333,0.4)	(0.266667,0.333333,0.4)
A_2	(0.635,0.13,0.235)	(0.266667,0.333333,0.4)	(0.266667,0.333333,0.4)
A_3	(0.845,0.1,0.055)	(0.266666,0.333331,0.400003)	(0.266667,0.333333,0.4)
A_4	(0.745,0.1,0.155)	(0.266666,0.333331,0.400003)	(0.266666,0.333334,0.4)
A_5	(0.855,0.08,0.065)	(0.266661,0.333333,0.400006)	(0.266667,0.333333,0.4)
A_6	(0.885,0.07,0.045)	(0.266657,0.333358,0.399985)	(0.266667,0.333333,0.399999)
A_7	(0.24,0.14,0.14)	(0.266666,0.333331,0.400003)	(0.266667,0.333333,0.4)

Table 4. Decision matrix (cont.)

	C_4	C_5
A_1	(0.309908,0.327804,0.362287)	(0.5,0.4,0.1)
A_2	(0.318261,0.328696,0.353043)	(0.2,0.5,0.3)
A_3	(0.317291,0.327986,0.354724)	(0.8,0.1,0.05)
A_4	(0.286391,0.315756,0.397853)	(0.5,0.4,0.1)
A_5	(0.313318,0.327742,0.35894)	(0.8,0.1,0.05)
A_6	(0.310726,0.326944,0.36233)	(0.5,0.4,0.1)
A_7	(0.16458,0.239117,0.596303)	(0.1,0,0.9)

Step 2. Determining the weight of the criteria

From Eq.(7), we get the weights w_j of criteria C_j are $w_1 = w_2 = w_3 = 0.21, w_4 = 0.19, w_5 = 0.18$.

Step 3. Determining the perfect choice

The perfect choice is

$A_b = (A_b(1), A_b(2), A_b(3), A_b(4), A_b(5))$

where $A_b(1) = A_b(2) = A_b(5) = (1, 0, 0)$ and $A_b(3) = A_b(4) = (0, 0, 1)$.

Step 4. Calculating the dissimilarity measure of each alternative to the perfect choice

The dissimilarity measure of each alternative and the perfect choice is calculated by Eq.(8) (Table 5).

$DIS_E(A_1, A_b) = 0.325$, $DIS_E(A_2, A_b) = 0.3719$, $DIS_E(A_3, A_b) = 0.2848$,
$DIS_E(A_4, A_b) = 0.3341$, $DIS_E(A_5, A_b) = 0.2839$, $DIS_E(A_6, A_b) = 0.3139$,
$DIS_E(A_7, A_b) = 0.4383$.

Step 5. Ranking the alternatives

We use Eq.(9) to rank the alternatives based on the dissimilarity measure of each alternative and the perfect choice

$A_7 \prec A_2 \prec A_4 \prec A_1 \prec A_6 \prec A_3 \prec A_5$

This result shows that alternative A_5 (Public parks watering (PPW)) is the best choice (Table 5).

Table 5. Ranking of alternatives

Alternatives	$DIS_E(A_i, A_b)$	Rank
TF	0.3250	4
VW	0.3719	6
FW	0.2848	2
AI	0.3341	5
PPW	0.2839	1
GCW	0.3139	3
DW	0.4383	7

If we consider the same weight for all criteria ($w_j = 0.2, j = 1,2, \ldots, 5$), we have the results as shown in Table 6.

Table 6. Ranking of alternatives with the same weight for all criteria

Alternatives	$DIS_E(A_i, A_b)$	Rank
TF	0.3256	4
VW	0.3745	6
FW	0.2819	2
AI	0.3345	5
PPW	0.2810	1
GCW	0.3150	3
DW	0.4405	7

Table 7. Ranking of the alternatives with different weight vectors

Alternatives	$w1 = (0.1,0.2,0.2,0.4,0.1)$		$w2 = (0.25,0.25,0.25,0.25,0)$		$w3 = (0,0.25,0.25,0.25,0.25)$	
	$DIS_E(A_i, A_b)$	Rank	$DIS_E(A_i, A_b)$	Rank	$DIS_E(A_i, A_b)$	Rank
TF	0.3623	4	0.3325	4	0.3752	4
VW	0.3855	6	0.3556	6	0.4123	6
FW	0.3394	1	0.3247	3	0.3274	1
AI	0.3703	5	0.3437	5	0.3781	5
PPW	0.3395	2	0.3237	2	0.3279	2
GCW	0.3569	3	0.3139	1	0.3751	3
DW	0.4461	7	0.4262	7	0.4430	7

Table 8. Comparing the ranking results of our method and the ranking results of Pan *et al.* (2018) with the same weight for all the criteria

Alternatives	Rank				
	Our method	Pro-economy	Pro-social	Pro-environment	WRAE with a generalized parameter
TF	4	5	5	5	5
VW	6	6	6	6	6
FW	2	2	1	1	1
AI	5	4	4	3	4
PPW	1	1	2	2	2
GCW	3	3	3	4	3
DW	7	7	7	7	7

Now, we give examples of results using our method with the different weight vectors. For instance, with $w1$ we considered human health risk criteria more important than others; with $w2$ we ignored the government policy criteria; and with $w3$ we dismissed the public acceptability criteria. These results are shown in Table 7. Finally, we also recalled the results cited in Pan *et al.* (2018) in Table 8.

Conclusions

In this paper, we introduced a new dissimilarity measure (in Eq.(1)). After that, we introduced a MCDM using the dissimilarity measure of picture fuzzy sets. Finally, we applied the proposed method to evaluate water reuse applications. When the weights changed, i.e. the priority for the criteria changed, the results also changed. In Pan *et al.* (2018), the authors used the hesitation of the fuzzy soft sets and combined this with the score function of them to evaluate the water reuse applications under uncertainty. This is the complexity of the methods of Pan *et al.* (2018). By characterizing the data of the water reuse applications in Pan *et al.* (2018), we find that the use of picture fuzzy sets can be applied to this problem. Our method represents a new approach to this problem and the calculation is simpler than Pan's. In the future, we plan to further apply this method to other problems as well as to study new cities to apply this method to help resolve practical problems.

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References

Atanassov K. T. (1986). Intuitionistic fuzzy sets. Fuzzy sets and Systems. Vol 20 (1). pp. 87-96.

Bhutia P. W. and Phipon R. (2012). Application of AHP and TOPSIS method for supplier selection problem. IOSR Journal of Engineering. Vol 2 (10). pp. 43-50.

Boran F. E., Genç S., Kurt M. and Akay D. (2009). A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Systems with Applications. Vol 36 (8). pp. 11363-11368.

Cuong B. C. and Kreinovich V. (2013). Picture Fuzzy Sets-a new concept for computational intelligence problems. In the 3rd World Congress on Information and Communication Technologies (WICT'2013), December 15-18 2013, Hanoi, Vietnam. pp. 1-6.

Dinh N. V., Thao N. X. and Chau N. M. (2015). On the picture fuzzy database: theories and application. Journal of Science and Development. Vol 13 (6). pp. 1028-1035.

Dinh N. V., Thao N. X. and Chau N. M. (2017). Some dissimilarity measures of picture fuzzy set. In the 10th Fundamental and Applied IT Research (FAIR'2017), August 17-18, 2017, Danang, Vietnam. pp. 104-109.

Hoa N. D. and Thong P. H. (2017). Some Improvements of Fuzzy Clustering Algorithms Using Picture Fuzzy Sets and Applications for Geographic Data Clustering. VNU Journal of Science: Computer Science and Communication Engineering. Vol 32 (3). pp. 32-38.

Jadidi O., Firouzi F. and Bagliery E. (2010). TOPSIS method for supplier selection problem. World Academy of Science, Engineering and Technology. Vol 47. pp. 956-958.

Kavita, Yadav S. P. and Kumar S. (2009). A multi-criteria interval-valued intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Lecture Notes in Computer Science. Vol 5908. pp. 303-312.

Maldonado-Macías A., Alvarado A., García J. L. and Balderrama C. O. (2014). Intuitionistic fuzzy TOPSIS for ergonomic compatibility evaluation of advanced manufacturing technology. The International Journal of Advanced Manufacturing Technology. Vol 70 (9-12). pp. 2283-2292.

Miller G. W. (2006). Integrated concepts in water reuse: managing global water needs. Desalination. Vol 187. pp. 65-75.

Omorogbe D. E. A. (2016). A review of intuitionistic fuzzy topsis for supplier selection. AFRREV STECH: An International Journal of Science and Technology. Vol 5 (2). pp. 91-102.

Pan Q., Chhipi-Shrestha G., Zhou D., Zhang K., Hewage K. and Sadiq R. (2018). Evaluating water reuse applications under uncertainty: generalized intuitionistic fuzzy-based approach. Stochastic Environmental Research and Risk Assessment. Vol 32 (4). pp. 1099-1111.

Pérez-Domínguez L., Alvarado-Iniesta A., Rodríguez-Borbón I. and Vergara-Villegas O. (2015). Intuitionistic fuzzy MOORA for supplier selection. Dyna. Vol 82 (191). pp. 34-41.

Solanki R., Gulati G., Tiwari A. and Lohani Q. M. D. (2016). A correlation based Intuitionistic fuzzy TOPSIS method on supplier selection problem. In 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), July 24-29, 2016, Vancouver, Canada. pp. 2106-2112.

Son L. H. (2015). DPFCM: A novel distributed picture fuzzy clustering method on picture fuzzy sets. Expert Systems with Applications. Vol 42. pp. 51-66.

Son L. H. (2017). Measuring analogousness in picture fuzzy sets: from picture distance measures to picture association measures. Fuzzy Optimization and Decision Making. Vol 16 (3). pp. 359-378.

Yayla A. Y., Yildiz A. and Özbek A. (2012). Fuzzy TOPSIS method in supplier selection and application in the garment industry. Fibres and Textiles in Eastern Europe. Vol 4 (93). pp. 20-23.

Yildiz A. and Yayla A. Y. (2015). Multi-criteria decision-making methods for supplier selection: A literature review. South African Journal of Industrial Engineering. Vol 26 (2). pp. 158-177.

Zadeh L. A. (1965). Fuzzy sets. Information and Control. Vol 8 (3). pp. 338-353.

Zarghami M. and Szidarovszky F. (2009). Stochastic-fuzzy multi criteria decision making for robust water resources management. Stochastic Environmental Research and Risk Assessment. Vol 23. pp. 329-339.

Zeng S. and Xiao Y. (2016). TOPSIS method for intuitionistic fuzzy multiple-criteria decision making and its application to investment selection. Kybernetes. Vol 45 (2). pp. 282-296.